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THE THEORY OF RELATIVITY.

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SCOTT describes the noble Saracen as receiving with politely concealed disbelief the Crusader's statement that water, when cold enough, would become so solid that an army could march over it; doubtless the keenest minds of the Middle Ages would have met with incredulity the assertion that the dimensions of bodies change with their temperature. The newer theories of matter and mass and the results of radioactivity have only recently torn us from the moorings of beliefs which had come to be regarded as no less immutable than eternal truth, and have left us in a receptive mood for any changes in physical theory, no matter how startling. Nevertheless, it is with considerable of a shock to our conservatism that we accept some of the conclusions of the new theory of relativity, that system of physical theory which has been developed within the past few years, and is to-day accepted by many of the keenest minds among the physicists of the world, some going so far as to call it the greatest advance in physical theory since the days of NEWTON.

For, taken by themselves, some of its conclusions seem no less than revolutionary. The dimensions of a body are affected by the rate of its motion through space, and not only are the units of length thus affected, but also those of mass and time; a clock rated to keep a certain time in a stationary system would change its rate in a moving system. No velocity in this theory can exceed that of light, i. e., this is infinity as far as velocities are concerned. Theoretically, in the theory of relativity, a swiftly rotating disk could not keep its shape as

a disk, for, because of its speed of rotation, its circumference must shorten while its radius remains the same; for great velocities it should warp to an epicycloidal cross-section. The mass of a body is a measure of its energy content; if a body radiates an amount of energy L, its mass diminishes by the amount L/c^2, where c is the velocity of light; energy and mass would then be as equivalent magnitudes as heat and mechanical energy; if the theory of relativity is true, Newtonian dynamics must be abandoned. More than this, no less an authority than POINCARÉ has stated the possibility that under this principle gravitation may be propagated with the velocity of light, instead of with a velocity nearly, if not absolutely infinite, as at present held, bringing the principle of gravitation from its present mysterious isolation into kinship with light and electricity.

It is evident that a physical theory with such possibilities as this may conceivably have many points of interest for the astronomer. In any event, the attention which it has been and is attracting is sufficient reason for its consideration here. The past histories of physical theories thought at one time to be supported by the full testimony of experiment and rigorous mathematics is sufficient to cause some conservatism, but the names of those who have accepted the theory form some guarantee of its value, and it is always well to remember in this connection the statement so well put by Sir J. J. THOMPSON that "a physical theory is to be regarded as a policy, rather than as a creed."

In 1881 and again in 1887, with greater precautions against error, MICHELSON performed what has since become one of the most famous experiments in modern physics. The question as to the relative movement of matter and the æther had long been a puzzling one. Theory and experiment have alike shown that no effect of the first order due to the relative movement of æther and matter was to be expected in optical phenomena, but there should theoretically be an effect of the second order, that is, a term of the order v^2/c^2, where v is the velocity of the moving matter and c the velocity of light. MICHELSON'S apparatus was so arranged as to determine by means of interference this residual effect of the second order; the result,

however, was negative. As an illustration of the delicacy of the experiment and the precautions which must be taken, I may here mention that the experiment has recently been discussed by LÜROTH to find whether the centrifugal force set up by the Earth's rotation, in connection with the known results of the theory of elasticity, could cause a sufficient change of length in the parts of MICHELSON's apparatus to account for his negative result. LÜROTH finds that a change in length of a meter rod in the experiment due to the change in centrifugal force as the apparatus was rotated so as to lie in the direction of the motion through space and then at right angles to this, was slightly over .001μ, while to explain the negative result secured a change of about .005μ would be required.

An explanation of MICHELSON's negative result was independently suggested by FITZGERALD and by LORENTZ without regard to the then undeveloped theory of relativity, namely that the negative result could be explained by the hypothesis that the dimensions of the apparatus used actually changed by enough due to its own motion through space to counteract and balance any shift of interference fringes. The required change in length is an exceedingly small one; in the case of a similar effect on the dimensions of the Earth it would require that the diameter of the Earth in a direction parallel to its direction of motion be shortened only two and a half inches.

Other experiments have been made to detect the influence of the Earth's motion on optical phenomena, of which RAYLEIGH's and BRACE's to find a double-refraction effect due to change of refractive index are the most noted; but all such experiments have been similarly negative as to result.

Suggested in part by the need for a physical system of laws which should conform to modern electrical theories of matter and which should explain these negative results, the theory of relativity has been developed, mainly by Professor EINSTEIN of Zürich. Its list of adherents is a long one, with such names as those of LORENTZ and POINCARÉ at the head; objections have been raised to it by MICHELSON, LARMOR, and others, but it may fairly be said to be a widely accepted theory to-day.

Although it has been developed with a rigorous regard to the electro-magnetic theory of matter, it is possible to state

some of its conditions quite simply. EINSTEIN states it: "The laws of nature are independent of the conditions of motion of a given system, at least in case the motion of the system is free from acceleration."

It may instead be put in the form of the postulates:

(1) All motion is relative.

(2) The velocity of light is independent of the motion either of source or observer.

Of these postulates the first is self-evident and universally accepted. The second is less so, and more difficult to understand. We are accustomed to assume without question that the velocity of light which we measure is the same, whatever may be the motion of the source of the light, though COMSTOCK has suggested as "A Neglected Type of Relativity" a hypothesis that this may not be true. But that the velocity of light will come out the same if the observer is moving toward or away from the source is less easy to accept. That this should be true involves the following changes in the units of space, time, etc., in the moving system: Letting M_0, L_0, and T_0 denote the units of mass, space and time for a body at rest, then, when the same body is in motion with a velocity v, the velocity of light being c, the following equations will connect the units of the moving system with the absolute units of the body when at rest:—

$$M = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$L = L_0$ for cross motion

$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$ for motion in direction of L

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

While some of the above ratios can be derived geometrically, any one who appreciates a beautifully worked out and clear-cut mathematical treatment will refer to the original papers by EINSTEIN, or to WIECHERT'S excellent résumé of the theory, where the system is built up from the simple fundamental

assumption and carried up through to the electro-magnetic theory of light and the electronic theory of matter. The preliminary equations are comparatively simple.

Following EINSTEIN, let S and S^1 be two systems with similar standards of length and synchronous clocks, providing these are compared when the two systems are at rest relatively to each other. In case S and S^1 are relatively at rest, any law of nature which holds for one system must hold for the other. The principle of relativity extends this complete coincidence also to the case that S^1 is in a uniform state of motion with regard to S. In particular, the velocity of light must be the same in both systems. Using the variables x, y, z, t, for the system S and x^1, y^1, z^1, t^1 for the system S^1, for simplicity choosing the position of the x^1-axis so as to coincide with the direction of motion of S^1, which is moving with a velocity v, the transformation equations will be of the form—

$$x^1 = a\,(x - vt)$$

$$y^1 = by$$

$$z^1 = cz$$

Since the velocity of light must be the same for both systems, both of the following equations must hold:

$$x^2 + y^2 + z^2 = c^2\,t^2$$

$$x^{12} + y^{12} + z^{12} = c^2\,t^{12}$$

Then, by a simple reduction the transformation equations must take the form—

$$t^1 = \phi\,(v)\,\beta\,(t - \frac{v}{c^2}\,x)$$

$$x^1 = \phi\,(v)\,\beta\,(x - vt)$$

$$y^1 = \phi\,(v)\,y$$

$$z^1 = \phi\,(v)\,z, \text{ where } \beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For this case it can be shown that $\phi\,(v) = 1$, and the equations take the form—

$$t^1 = \beta\,(t - \frac{v}{c^2}\,x)$$

$$x^1 = \beta\,(x - vt)$$

$y^1 = y$

$z^1 = z$

where $\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

This transformation has received the name of LORENTZ-transformation and is of great importance. It has been shown to be the only transformation which leaves the laws of the electromagnetic theory unchanged.

Let the coördinates of two points in the moving system S^1 be x^1_1, y^1_1, z^1_1, and x^1_2, y^1_2, z^1_2. Between the coördinates of these points referred to the reference system S at any time t of S from the above transformation the following relations hold:

$$x_2 - x_1 = \sqrt{1 - \frac{v^2}{c^2}}\ (x^1_2 - x^1_1)$$

$$y_2 - y_1 = y^1_2 - y^1_1$$

$$z_2 - z_1 = z^1_2 - z^1_1$$

We have, then, a shortening in the direction of relative motion equal to $1 : \sqrt{1 - \frac{v^2}{c^2}}$. A relative motion with a velocity greater than that of light is then impossible on this theory, for then the quantity under the radical sign would become imaginary. Similarly a clock on S^1 would keep slower time in the ratio $\sqrt{1 - \frac{v^2}{c^2}} : 1$ than the same clock would in a system at rest, though the observer on the moving system would not be conscious of such a difference, nor of the fact that his standards of length differed from those on a system at rest. A clock at the equator would lose over a clock at the pole because of its relative motion, though the difference would be infinitesimal. Even in the case of the motion of the Earth about the Sun the ratio of v to c is only of the order of 0.0001.

Let a point in the moving system S^1 move uniformly according to the conditions—

$x^1 = u^1_x\ t^1$

$y^1 = u^1_y\ t^1$

$z^1 = u^1_z\ t^1$

Transforming and referring to the reference system S:—

$$u_x = \frac{u^1_x + v}{1 + \frac{vu^1_x}{c^2}}$$

$$u_y = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vu_x}{c^2}} u^1_y$$

$$u_z = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vu^1_x}{c^2}} u^1_z$$

The law of the parallelogram of velocities then holds only to a first approximation. Placing

$$u^2 = u^2_x + u^2_y + u^2_z$$

$$u^{12} = u_x^{12} + u_y^{12} + u_z^{12}$$

and α for the angle between the x^1-axis (v) and the direction of motion of the point in reference to S^1

$$u = \frac{\sqrt{(v^2 + u^{12} + 2vu^1 \cos\alpha) - \left(\frac{vu^1 \sin\alpha}{c^2}\right)^2}}{1 + \frac{vu^1 \cos\alpha}{c^2}}$$

and if both velocities v and u^1 are in the same direction, this reduces to:—

$$u = \frac{v + u^1}{1 + \frac{vu^1}{c^2}}$$

This is the famous addition theorem of relativity to which some physicists have taken objection. From this it follows that the addition of the velocity u^1 in a moving system to the velocity v of the system does not give $u^1 + v$, as one would suppose, but $\frac{v + u^1}{1 + \frac{vu^1}{c^2}}$. It also follows from this equation that the addition of two velocities smaller than c will never give a velocity greater than c, and also that the addition of

another velocity less than that of light to this velocity, c, will again give a velocity only equal to that of light. This can be easily shown as follows: Let the two velocities be represented by $c - n$ and $c - m$; substituting in the addition theorem and reducing, $c \frac{2c - n - m}{2c - n - m + \frac{nm}{c}}$, which is always less than c.

The only physical velocities with which we are acquainted of sufficient magnitude to render it possible to test the truth of the theory are those of the β-rays which range from one third to nine tenths of the velocity of light.

As to the consequences of the theory of relativity from the standpoint of the astronomer, but little is to be said. De Sitter finds that under the theory of relativity the deviation from ordinary Keplerian motion is periodic, but this deviation, in terms of the Earth's radius, can amount only to one hundred-millionth, a quantity entirely insensible.

Also on this theory, even a planet of infinitesimal mass will show a movement of the perihelion; de Sitter finds the following motions of perihelia:

Mercury	7″.15	per	century
Venus	1 .42	"	"
Earth	0 .63	"	"
Comet Encke	0 .30	"	"
Comet Halley	0 .007	"	"

The only effect which reaches an appreciable amount is the motion of the perihelion of Mercury. Unfortunately this quantity just represents the well-known excess of observation over theory, explained by Seeliger as due to the attraction of the mass forming the zodiacal light. It is therefore impossible at present to decide which is the true cause. There is no other effect due to relativity which can be determined astronomically at present.

We may sum up as being against the theory:

Some physicists find the addition theorem difficult of acceptance.

Except as noted below, it does not appear possible at present to prove the theory one way or the other.

In support of the theory we may give the following considerations:

It unifies many physical facts, particularly in the theory of moving matter from the standpoint of the electromagnetic theory. LORENTZ speaks of the theory as follows: "His (EINSTEIN'S) results agree in the main with those which we have obtained . . . the chief difference being that EINSTEIN simply postulates what we have derived, with some difficulty and not entirely satisfactorily, from the fundamental equations of the magnetic field. By doing so, he may certainly take credit for making us see in the negative results of experiments like those of MICHELSON, RAYLEIGH and BRACE, not a fortuitous compensation of opposing effects, but the manifestation of a general and fundamental principle. . . . It would be unujst not to add that, besides the fascinating boldness of its starting point, EINSTEIN'S theory has another marked advantage over mine. Whereas I have not been able to obtain for the equations referred to moving axes *exactly* the same form as for those which apply to a stationary system, EINSTEIN has accomplished this by means of a system of new variables slightly different from those which I have employed. . . . Recent experiments by BUCHERER on the electric and magnetic deflections of the β-rays, made after a method that permits greater accuracy than could be reached by KAUFMANN have confirmed the formula for the variation of the transverse electromagnetic mass with the velocity, so that in all probability the only objection that could be raised against the hypothesis of the deformable electron and the theory of relativity has now been removed."

The theory, then, is at variance with no known experiments or facts.

It gives a satisfactory explanation for the negative results secured by MICHELSON and others.

It is directly supported by KAUFMANN and BUCHERER'S experimental proof that the transverse mass of a moving electron is a function of its velocity.

It may possibly afford a theory of gravitation which shall bring this force into the realm of the other physical forces moving with the velocity of light.

On this point we may quote POINCARÉ: "It is important to examine this hypothesis more closely and, in particular, to seek what modification it would oblige us to make in the laws of gravitation. This is what I have sought to determine: I was at first led to suppose that the propagation of gravitation is not instantaneous, but moves with the velocity of light. This seems to be in contradiction to a result obtained by LAPLACE, who announced that this velocity of propagation, if not instantaneous, is at least much more rapid than light. But, in reality, the question which LAPLACE put differs much from that which we are treating. With LAPLACE, the introduction of a finite velocity of propagation was the *only* modification which he made in NEWTON'S laws. Here, on the contrary, this modification is accompanied by numerous others; it is then possible, and in effect occurs, that there is produced a partial compensation. . . . I have investigated whether it would be possible to determine these functions (of the components of attraction) in such a way that if they were affected by a LORENTZ transformation they would conform to the ordinary law of gravitation, supposing that the velocities are small enough so that their squares can be neglected in comparison with the square of the velocity of light. The answer is affirmative; the divergence from the ordinary law of gravitation is of the order of $\frac{v^2}{c^2}$; whereas if one postulates only, as did LAPLACE, that the velocity of propagation is that of light, this divergence would be of the order of $\frac{v}{c}$ (where v is the velocity of the planet), that is, 10,000 times greater. It is then not absurd, *a priori,* to suppose that astronomical observations are not sufficiently precise to discover a divergence as small as that which we imagine. But only a profound discussion would allow a decision on this point."

The literature of the subject is already very extensive, and there are few volumes of the physical journals during the past six years which do not contain articles bearing more or less closely on the theory of relativity. Some of the more important articles are referred to in the following partial bibliography:

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